Boundary Element Analysis of a Trapezoidal Transmission Line

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Abstract—**A transmission line with a trapezoidal cross section is analyzed using the boundary element method (BEM) and the quasi-static approximation. By utilizing a convenient choice for the fundamental solution, the efficiency of the method is clearly established. The analysis is verified by comparisons with results in the literature and measured data.**

I. Introduction

As the desired operating frequency of transmission lines increases, the conductor losses also increase. It is, therefore, paramount that the currents on the metal surfaces bereduced. Recently, several structures have been designed to achieve this goal [1]. One possible candidate is a transmission line with a trapezoidal cross section, which is shown in Fig. 1.

The trapezoidal geometry does not lend itself easily to most conventional methods of analysis. The finite-element method (FEM) may be used (e.g., [2]), although with some difficulty since this is an open structure. The boundary-element method (BEM) may also be used ([3]-[5]), and with advantage. By a proper choice for the fundamental solution, (i.e., Greens function), the computation can be reduced to two line integrals over the contour Γ_{abcd} (for the infinitely thin strip). This greatly reduces the amount of computation involved, and is clearly superior to the FEM in this respect.

II. Formulation

For a transmission line with constant cross-section, the quasi-TEM approximation entails the solution of Laplace's equation in two dimensions. The application of the BEM in such a situation is well known [6]. Essentially, the potential on the boundary of a region R is given by

$$c_i u(x_i, y_i) + \int_\Gamma u q^* \, d\Gamma = \int_\Gamma q u^* \, d\Gamma, \qquad (1)$$

where $u(x_i, y_i)$ is the desired potential on the boundary, $q = \partial u / \partial n$, with $\hat{n}$ the unit normal out of the boundary, and c_i is a constant determined by the smoothness of the boundary. The contour Γ depends on which region is enclosed. For Region 1, Γ consists of the path along the groundplane from point a to infinity, Γ_∞, the path along the groundplane from

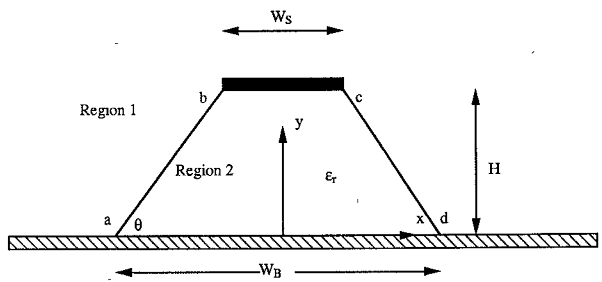

Fig. 1. Layout of the trapezoidal transmission line. Regions 1 and 2 consist of homogeneous dielectrics $\epsilon_{r1}\epsilon_0$ and $\epsilon_{r2}\epsilon_0$, respectively.

infinity back to d, and the path $abcd$. The contour for Region 2 is the is identical, but excludes the path Γ_∞. The corresponding fundamental solution and normal derivative are u^* and q^*, respectively. Since both u and u^* are zero, the contour at infinity can be eliminated, and the ground plane path is eliminated for the left-hand side integral. The remaining task is to choose a fundamental solution u^* that also obeys the boundary conditions on the ground plane, thereby rendering

$$c_i u(x_i, y_i) + \int_{\text{abcd}} u q^* \, d\Gamma = \int_{\text{abcd}} q u^* \, d\Gamma \qquad (2)$$

This is accomplished by solving

$$\nabla^2 u^* = \delta(x - x')\delta(y - y'),$$

for the homogeneous half space $y \geq 0$ (i.e., Fig. 1) and then applying the method of images, yielding

$$u^*(x, y; x', y') = \frac{-1}{2\pi} \ln \left[\sqrt{\frac{(x - x')^2 + (y - y')^2}{(x - x')^2 + (y + y')^2}} \right]. \qquad (3)$$

It is clear that (3) will yield the desired result on the ground plane. Equation (2) is then discretized for Regions 1 and 2 using constant basis functions, and the following boundary conditions are applied for the contour segment bc:

$$u_i^1 = u_i^2 = V.$$

The interface conditions along segments ab and cd are

$$u_i^1 = u_i^2 \text{ and } \epsilon_{r1} q_i^1 = -\epsilon_{r2} q_i^2.$$

The basis functions were applied on both equal and unequal intervals (i.e., finer towards the edges), with similar results in either case. The system of equations for the two regions are then combined and solved as in [6]. From the calculated

Manuscript received August 22, 1991. This work was supported in part by the U.S. Army Contract DAAL-03-88-K-0005 and in part by ATR Optical and Radio Communications Research Laboratories. Editorial decision made by S. Maas.

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IEEE Log Number 9104372.

TABLE I
CHARACTERISTIC IMPEDANCE OF MICROSTRIP WITH $\epsilon_r = 1.0$

$\frac{W_s}{H}$	0.1	1.0	2.0	10.0
Z (From [7]) (Ohms)	265	127	90	30
Z (From BEM in this letter)	264	127	90	30

values of the flux q on the strip, the capacitance, characteristic impedance and phase velocity can be calculated in standard fashion (e.g., [3]).

III. RESULTS

Since this structure has only recently been proposed, there are few established results in the literature for which comparisons can be made. With $\epsilon_{r1} = \epsilon_{r2} = 1.0$, a comparison with Wheeler [7] can be made, and the results are given in Table I. Convergence was tested by increasing the number of basis functions until the characteristic impedance variation was less than 0.25%. A plot of a typical convergence check is given in Fig. 2. Several trapezoidal lines have been designed and fabricated with different values of the structure parameters [8], all to achieve a 50 ohm characteristic impedance. A few of the combinations are shown in Table II, along with characteristic impedance values calculated by the BEM. The measured return loss is 20 db or greater, and the calculated values and fall well within this range.

IV. CONCLUSION

A new transmission line structure has been analyzed using the BEM. By a proper choice of the fundamental solution, this method is shown to be very efficient in analyzing this structure. Comparisons agree well with established results, and some experimentally verified design data is given.

ACKNOWLEDGMENT

The authors thank Dr. Ogawa for providing data from his measurements.

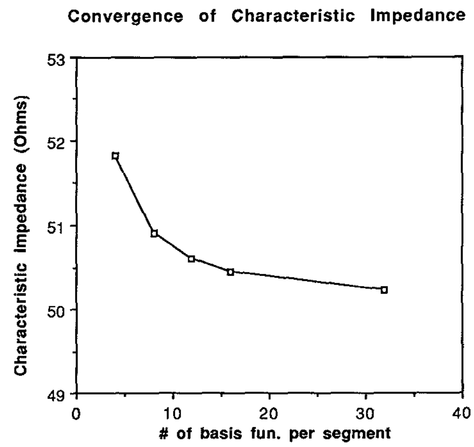

Fig. 2. Convergence of characteristic impedance vs. number of basis functions on each trapezoid segment (e.g., "ab" or "bc").

TABLE II
DESIGN OF 50 OHM TRANSMISSION LINE ($\epsilon_r = 3.3$)

$\frac{W_s}{H}$	2.4	2.4	2.5	2.5	2.5	2.6	2.6	2.7
θ (deg.)	20	24	30	35	45	60	75	90
Z (Ohms)	50.7	50.8	49.7	49.9	50.3	49.6	50.5	50.2

REFERENCES

[1] H. Ogawa, Hasegawa, S. Banba, and Nakamoto, "MMIC transmission lines for multilayered MMIC's," presented at *IEEE MTT-S Int. Microwave Symp.*, Boston, MA, June 10-14, 1991.
[2] Z. Pantic and R. Mittra, "Quasi-TEM analysis of microwave transmission lines by the finite-element method," *IEEE Trans. Microwave Theory Tech.*, vol. 34, no. 11, pp. 1096-1103, Nov. 1986.
[3] M. Ikeuchi, "Boundary element analysis of shielded microstrip lines with dielectric layers," *Trans. IECE Jap.*, vol. E-67, no. 11, pp. 585-590, Nov. 1984.
[4] T. Honma, H. Itoh, and I. Fukai, "Application of pseudo-infinite boundary elements in a boundary element method to solve unbounded regions of open striplines," *Trans. IECE Jap.*, vol. E-66, no. 5, pp. 325-326, May 1983.
[5] T. Chang and C. Tan, "Analysis of a shielded microstrip line with finite metallization thickness by the boundary element method," *IEEE Trans. Microwave Theory Tech.*, vol. 38, no. 8, pp. 1130-1132, Aug. 1990.
[6] C. A. Brebbia and S. Walker, *Boundary Element Techniques in Engineering*. Newnes-Butterworths, 1980.
[7] H. Wheeler, Microwave Engineers Handbook, Horizon-House Microwave Inc., 1971, vol. 1, p. 137.
[8] H. Ogawa, private communication.